TESLA Report 2003-06

Tomasz Czarski, Ryszard Romaniuk, Krzysztof Poźniak
ELHEP Group, ISE, Warsaw University of Technology
Stefan Simrock
TESLA, DESY, Hamburg

Cavity Control System
Advanced Modeling and Simulations
For TESLA Linear Accelerator

ABSTRACT

The cavity control system for the TESLA - TeV–Energy Superconducting Linear Accelerator project is initially introduced. The elementary analysis of the cavity resonator on RF (radio frequency) level and low level frequency with signal and power considerations is presented. For the field vector detection the digital signal processing is proposed. The electromechanical model concerning Lorentz force detuning is applied for analyzing the basic features of the system performance. For multiple cavities supplied by one klystron the field vector sum control is investigated. Simulink model implementation is developed to explore the feedback and feed-forward system operation and some experimental results for signals and power considerations are presented.

1. INTRODUCTION

The TESLA concept is based on nine-cell superconducting niobium resonators called cavities to accelerate electrons and positrons. The acceleration structure is operated in standing π-mode wave at frequency of 1,3 GHz. The RF oscillating field is synchronized with the motion of a particle moving with the velocity of light across the cavity (figure1).

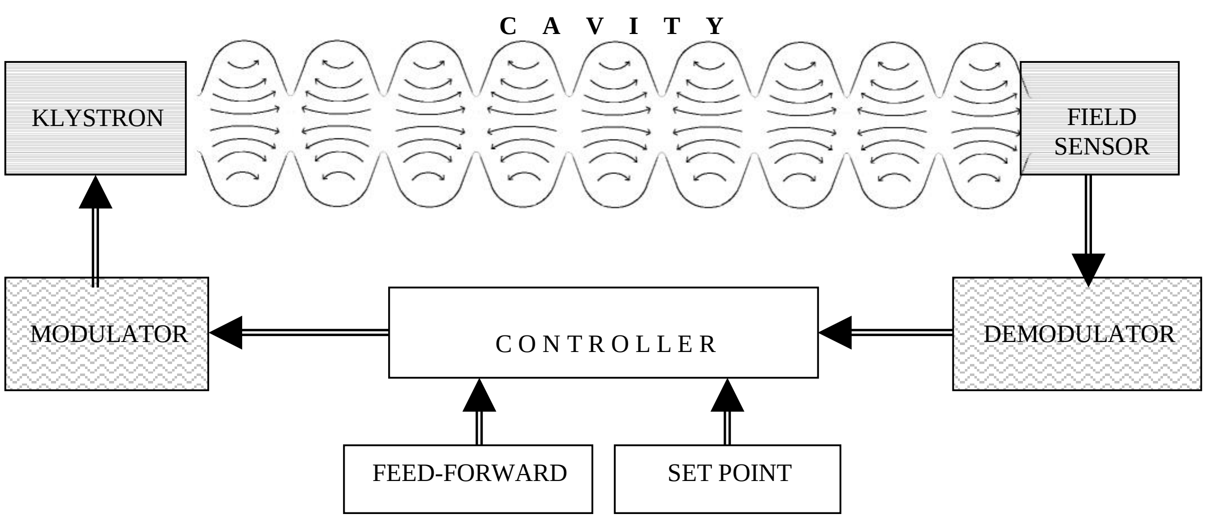

Figure 1. Schematic cavity field distribution and simplified block diagram of cavity control system.

The complex RF control system for the relativistic beam has been developed to cope with signal disturbances, non-linearity and with time-varying parameters, in order to stabilize the accelerating fields of the resonators. The control section, powered by one klystron, consists of four cryomodules containing eighth cavities each. One 10 MW klystron, through coupled wave-guide with circulator, supplies RF power to 32 cavities which are operated in pulsed mode with an average accelerating gradients of 25 MV/m. The control feedback system regulates the vector sum of pulsed accelerating fields in multiple cavities. The fast amplitude and phase control of the cavity field is accomplished by modulation of the signal driving the klystron. The digital controller stabilizes the detected real (in-phase) and imaginary (quadrature) components of the constant frequency incident wave according to the required set-point. Additionally the adaptive feed-forward is applied to improve compensation of repetitive perturbations induced by the beam loading and by the dynamic Lorentz force detuning (figure 1).

3. CAVITY RF LEVEL MODELING.

The signal relation for the cavity environment is obtained by applying signal-flow graph according to figure 2. The cavity is operated in 1.3 ms pulsed mode, 10 Hz rate; all quantities are represented in the ***Laplace transform*** space.

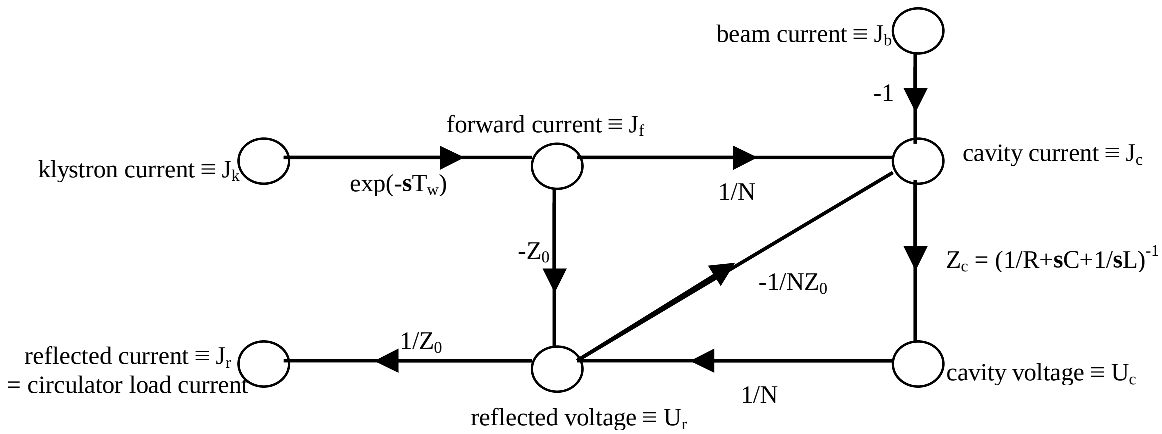

Figure 2. Signal-flow graph for cavity environment.

The klystron as a power amplifier is modeled by the RF current generator driving the wave-guide with circulator. The transmission line is parameterized by the impedance Z_0 and time delay T_w. The beam loading can be modeled as a current sink J_b feed by the cavity electromagnetic field. The bunched beam current has typically ~2 ps pulsed structure, 1 MHz rate and an average value of 8 mA. The cavity is represented as the resonant LCR circuit with impedance Z_c coupled to the wave-guide. The forward wave drives the coupler, which converts the signals according to the transformation ratio 1:N. Superposition of the output coupler signal and the beam loading results in the cavity current J_c and related by impedance Z_c cavity voltage U_c. The reflected wave is isolated by circulator and absorbed by the load matched to the wave-guide and therefore is no feedback to the klystron output.

Applying an equivalent parallel resistance connection	$R_L \equiv N^2Z_0 \,\|\, R$
and the cavity loaded impedance	$Z_L = (1/R_L + \mathbf{s}C + 1/\mathbf{s}L)^{-1}$
and substituting the current generator	$J_g \equiv J_f/N$,
results in the reflected voltage	$U_r = (U_c - J_gR_L)/N$
and the cavity voltage	$U_c = Z_L\cdot(2J_g - J_b) = Z_L\cdot J$.

Therefore the cavity can be represented by the transfer function in the corresponding forms:

$$Z_L(\mathbf{s}) = (1/R_L+ \mathbf{s}C+1/\mathbf{s}L)^{-1} = \omega_0\cdot\rho/(\omega_0/Q_L + (\mathbf{s}^2 + \omega_0^2)/\mathbf{s}) = \omega_{1/2}\cdot R_L/(\omega_{1/2} + (\mathbf{s}^2 + \omega_0^2)/2\mathbf{s},$$

where the following cavity parameters are applied: resonance pulsation ≡ $\omega_0 = 2\pi f_0 = (LC)^{-½}$, characteristic resistance ≡ $\rho = (L/C)^{-½}$, loaded quality factor ≡ $Q_L = R_L/\rho$, half-bandwidth (HWHM) ≡ $2\pi f_{1/2} = \omega_{1/2} = 1/2CR_L = \omega_0/2Q_L$.

4. ANALYTICAL SIGNAL AND (I, Q) MODELING.

According to the RF generator constant pulsation ω_g, and due to a narrow resonator bandwidth, the cavity voltage can be modeled in ***time domain*** as a *generalized oscillation*

$$u_r = A(t)\cdot\cos(\omega_g t + \varphi(t)) \quad \text{with its } \pi/2 \text{ shifted } \textit{Hilbert transform} \quad u_i = A(t)\cdot\sin(\omega_g t + \varphi(t)),$$

where the amplitude A and the phase φ are time-varying components with a relatively narrow spectral range.
This ordered pair of complementary signals called *analytical signal* can be represented as a vector or phasor in complex domain

$$\mathbf{u} \equiv (u_r, u_i) \equiv u_r + \boldsymbol{i}u_i = A\cdot\exp(\boldsymbol{i}(\omega_g t + \varphi)).$$

The cavity control system proceeds with a low level of frequency (I, Q) components:
the real signal component in-phase $\equiv I = v_r = A\cos\varphi$
and the imaginary signal component quadrature $\equiv Q = v_i = A\sin\varphi$.

In-phase and quadrature components can be detected by the complex demodulation (down-conversion) as follows:

$$(I, Q) = \mathbf{v} \equiv (v_r, v_i) \equiv (A\cos\varphi, A\sin\varphi) = Ae^{\boldsymbol{i}\varphi} = \mathbf{u}\cdot\exp(-\boldsymbol{i}\omega_g t).$$

The cavity input current is modeled as well as an *analytical signal* $\mathbf{j} = 2\mathbf{j}_g - \mathbf{j}_b$ created by the complex modulation (up-conversion), where $\mathbf{j}_b$ deals with ω_g Fourier component of the pulse structure beam loading current.
According to the cavity transfer function modeling, the relation between current and voltage *analytical signal* is given in ***Laplace space***:

$$\mathbf{U}(\mathbf{s}) = Z_L(\mathbf{s})\cdot\mathbf{J}(\mathbf{s}) = Z_L(\mathbf{s})\cdot(2\mathbf{J}_g(\mathbf{s}) - \mathbf{J}_b(\mathbf{s})).$$

The signal relation is modeled by signal-flow graph according to figure 3. The beam loading current is extracted outside of the signal-flow graph according to (I, Q) components relation: $\mathbf{I}(\mathbf{s}) = 2\mathbf{I}_g(\mathbf{s}) - \mathbf{I}_b(\mathbf{s})$.

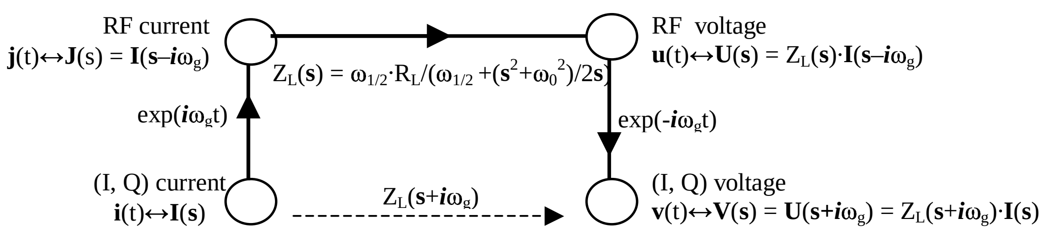

Figure3. Signal-flow graph for (I,Q) components and analytical signals.

The successive operations represented by the signal-flow graph in figure 3 yield the direct relation between the cavity input and output (I, Q) components as follows

$$\mathbf{V}(\mathbf{s}) = Z_L(\mathbf{s}+\boldsymbol{i}\omega_g)\cdot\mathbf{I}(\mathbf{s}), \text{ where}$$
$$Z_L(\mathbf{s}+\boldsymbol{i}\omega_g) = \omega_{1/2}\cdot R_L/(\omega_{1/2}+((\mathbf{s}+\boldsymbol{i}\omega_g)^2 + \omega_0^2)/2(\mathbf{s}+\boldsymbol{i}\omega_g)) = \omega_{1/2}\cdot R_L/(\omega_{1/2}+(\mathbf{s}(\mathbf{s}+2\boldsymbol{i}\omega_g)+(\omega_0-\omega_g)\cdot(\omega_0+\omega_g))/2(\mathbf{s}+\boldsymbol{i}\omega_g))$$

The resultant cavity transfer function can be effectively simplified for signals with a narrow spectral range relative to the generator frequency close to the cavity resonance pulsation, then for $|\mathbf{s}| << \omega_g \approx \omega_0$ yields

$$Z_L(\mathbf{s}+\boldsymbol{i}\omega_g) \approx \omega_{1/2}\cdot R_L/(\mathbf{s} + \omega_{1/2} - \boldsymbol{i}\Delta\omega) \equiv Z(\mathbf{s}), \text{ where cavity detuning } 2\pi\Delta f = \Delta\omega \equiv \omega_0 - \omega_g.$$

Therefore (I, Q) components relation can be written for phasor representation

$$(\mathbf{s} + \omega_{1/2} - \boldsymbol{i}\Delta\omega)\cdot\mathbf{V}(\mathbf{s}) = \omega_{1/2}\cdot R_L\cdot\mathbf{I}(\mathbf{s}).$$

Moving to ***time domain*** yields state space relation

$$d\mathbf{v}/dt = \mathbf{A}\cdot\mathbf{v} + \omega_{1/2}\cdot R_L\cdot\mathbf{i}$$

where $\mathbf{A} = (-\omega_{1/2}, \Delta\omega)$ for phasor representation or state matrix $\mathbf{A} = (-\omega_{1/2}, -\Delta\omega; \Delta\omega, -\omega_{1/2})$ for vector representation. Thus the resultant state space equation depends on the cavity bandwidth and detuning only.

The phasor solution of the state-space equation for the current step input $\mathbf{i}_0\cdot\mathbf{1}(t)$ is as follows

$$\mathbf{v}(t) = \mathbf{i}_0\cdot\omega_{1/2}\cdot R_L\cdot(1- \exp((-\omega_{1/2}+\boldsymbol{i}\Delta\omega)t))/(\omega_{1/2}- \boldsymbol{i}\Delta\omega) \qquad \text{for time } t \geq 0.$$

The cavity (I,Q) voltage step response for driving current i_0 = 32 mA (i_g=16 mA, i_b=0) is presented in figure 4 for the following parameters: loaded quality factor $Q_L = 3\cdot10^6$, characteristic resistance ρ =520 Ω (R_L = 1560 MΩ, $f_{1/2}$ = 217 Hz) detuning range Δf = 0 : 400 Hz.

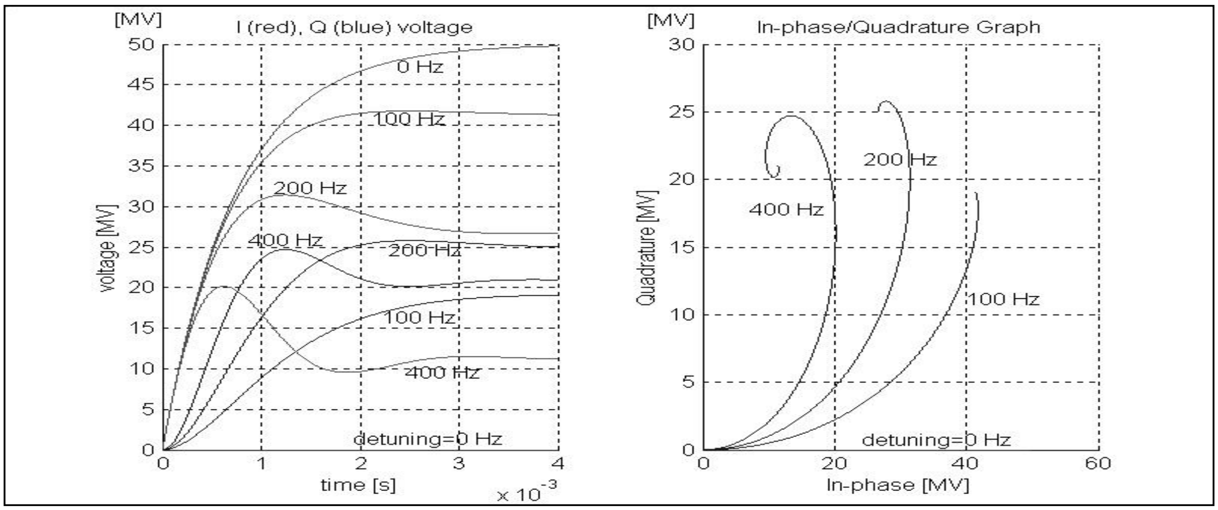

Figure 4. Cavity (I,Q) voltage step response for different detuning.

5. CAVITY VOLTAGE (I, Q) DETECTION.

In the TESLA control system only RF scalar signal is available and applied for detection cavity voltage (I, Q) components. In down-conversion operation the signal of 1300 MHz frequency is mixed with 1300.25 MHz local oscillation. The low-pass filter selects the signal of the difference 250 kHz intermediate frequency f_i preserving amplitude and phase information as follows

$$u(t) = u_r = A\cdot\cos(2\pi f_i\cdot t + \varphi).$$

Assuming a slow variation of the amplitude and phase for the period of delay $T = 1/4f_i = 1\mu s$, $\pi/2$ shifted copy $u_i = A\cdot\sin(2\pi f_i\cdot t + \varphi)$ completes an approximation of *analytical signal* $\mathbf{u}(t) \equiv (u_r, u_i) = A\cdot\exp(\boldsymbol{i}(2\pi f_i\cdot t+\varphi))$. Taking into account the measurement channel attenuation by factor 1/g and additional phase shifting $\Delta\varphi$ for an individual cavity the measured value is as follows

$$\mathbf{u}(t) = 1/g\cdot A\cdot\exp(\boldsymbol{i}(2\pi f_i\cdot t + \varphi+\Delta\varphi))$$

Therefore (I,Q) components can be obtained by complex demodulation according to relation

$$(I,Q) \equiv \mathbf{v}(t) = (v_r, v_i) = g\cdot\mathbf{u}(t)\cdot\exp(-\boldsymbol{i}(2\pi f_i\cdot t + \Delta\varphi)).$$

In the TESLA control system the digital signal processing is performed and the sequence of two $\pi/2$ shifted consecutive samples of the IF signal is applied to detect the discrete (I,Q) components as follows

$$(I,Q)_n \equiv \mathbf{v}(nT) = \mathbf{v}_n = g\cdot\mathbf{u}(nT)\cdot\exp(-\boldsymbol{i}(2\pi f_i\cdot nT+\Delta\varphi)) = g\cdot\exp(-\boldsymbol{i}\Delta\varphi)\cdot(-\boldsymbol{i})^n\cdot\mathbf{u}_n = \mathbf{k}_n\cdot\mathbf{u}_n,$$

where $\mathbf{u}_n = \mathbf{u}(nT) = (u(nT), u(nT\text{-}T)) = (u_n, u_{n-1})$ and coefficient $\mathbf{k}_n = g\cdot\exp(-\boldsymbol{i}\Delta\varphi)\cdot(-\boldsymbol{i})^n$ with period N=4.

Therefore four subsequent samples are as follow:

$$(I,Q)_n = g\cdot(u_n\cdot\cos(\Delta\varphi) + u_{n-1}\cdot\sin(\Delta\varphi);\ -u_n\cdot\sin(\Delta\varphi) + u_{n-1}\cdot\cos(\Delta\varphi));$$

$$(I,Q)_{n+1} = g\cdot(-u_{n+1}\cdot\sin(\Delta\varphi) + u_n\cdot\cos(\Delta\varphi);\ -u_{n+1}\cdot\cos(\Delta\varphi) - u_n\cdot\sin(\Delta\varphi));$$

$$(I,Q)_{n+2} = g\cdot(-u_{n+2}\cdot\cos(\Delta\varphi) - u_{n+1}\cdot\sin(\Delta\varphi);\ u_{n+2}\cdot\sin(\Delta\varphi) - u_{n+1}\cdot\cos(\Delta\varphi));$$

$$(I,Q)_{n+3} = g\cdot(u_{n+3}\cdot\sin(\Delta\varphi) - u_{n+2}\cdot\cos(\Delta\varphi);\ u_{n+3}\cdot\cos(\Delta\varphi) + u_{n+2}\cdot\sin(\Delta\varphi));$$

6. CAVITY POWER CONSIDERATION.

According to the *analytical signal* modeling, the cavity voltage and the current can be represented, after complex demodulation, as a relatively stable phasor or vector $\mathbf{v}(t) \equiv \mathbf{V}$ and $\mathbf{i}(t) \equiv \mathbf{I}$, each with its I, Q components. The real power is given in terms of complex voltage and conjugate current $P = ½\, Re\{\mathbf{V}\cdot\mathbf{I}^*\}$.

The forward power, which is provided by the wave-guide, reflects itself partly due to the mismatched input coupler and dissipates in the circulator load (figure 5). The residual transmitted power supplies cavity and feeds the beam loading. The objective of the accelerator system is to deliver power to the beam with the best efficiency.

Energy conservation yields $P_f = P_r + P_d + dW/dt + P_b$,
where

- forward power $\equiv P_f = ½\, Re\{\mathbf{V_f}\cdot\mathbf{I_f}^*\} = ½\,|\mathbf{I_f}|^2\cdot Z_0 \approx ½\,|\mathbf{I_g}|^2\cdot R_L$ ($R_L << R$)
- reflected power $\equiv P_r = ½\, Re\{\mathbf{V_r}\cdot\mathbf{I_r}^*\} = ½\,|\mathbf{I_r}|^2\cdot Z_0 \approx |\mathbf{V_c} - R_L\cdot\mathbf{I_g}|^2/2R_L$ ($R_L << R$)
- cavity dissipated power $\equiv P_d = ½\, Re\{\mathbf{V_c}\cdot\mathbf{I_c}^*\} = |\mathbf{V_c}|^2/2R \approx 0$
- electromagnetic energy stored in cavity $\equiv W = |\mathbf{V_c}|^2/2\rho\omega_0$,
- beam loading power $\equiv P_b = ½\, Re\{\mathbf{V_c}\cdot\mathbf{I_b}^*\} = |\mathbf{V_c}|\cdot I_{b0}\cdot\cos\varphi$

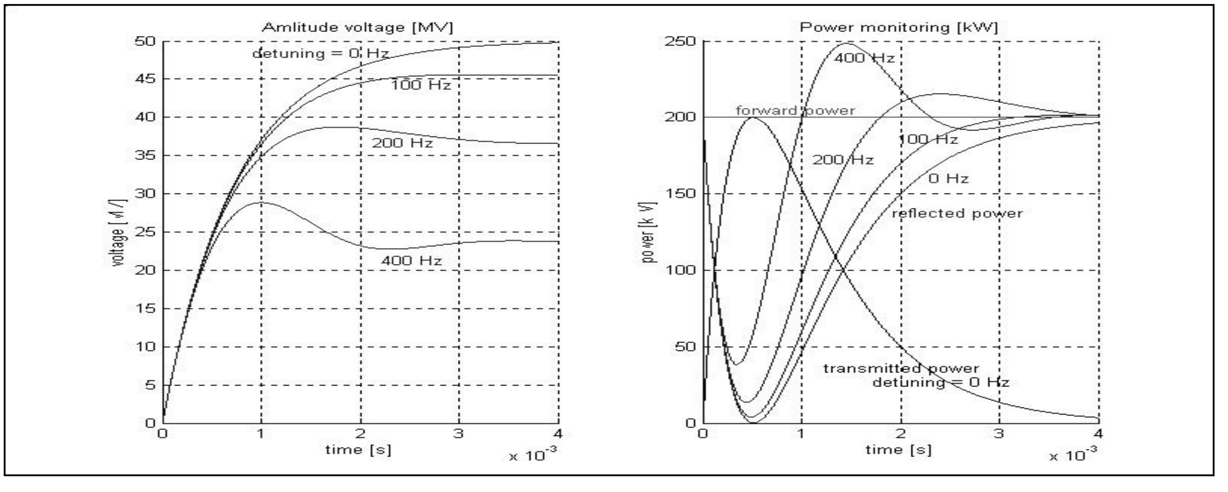

Figure 5. Cavity amplitude voltage step response and power monitoring for different detuning.

In a steady state with the beam loading and the detuned cavity ($\omega_g \neq \omega_0$) the main signals and parameters are as follows

- beam loading average current $\equiv I_{b0} = 8$mA – typical value
- RF component beam loading current $\mathbf{I_b} = |\mathbf{I_b}| = 2I_{b0}$ - determined as a reference phasor
- cavity required voltage $\mathbf{V_c} = |\mathbf{V_c}| \cdot e^{i\varphi}$ - $|\mathbf{V_c}|$, φ – determined, stabilized amplitude and phase
- generator current $\mathbf{I_g} = |\mathbf{I_g}| \cdot e^{i\theta}$ - $|\mathbf{I_g}|$, θ – amplitude and phase actuated by controller
- cavity actual voltage $\mathbf{V_c} = Z_L(i\omega_g)\cdot(2\mathbf{I_g} - \mathbf{I_b}) \approx 2R_L/(1 - i\Delta\omega/\omega_{1/2})\cdot(\mathbf{I_g} - I_{b0})$.

Equating the required and the actual cavity voltage $\mathbf{V_c}$, yields the stabilization equation. Solving this equation, the required forward power P_f is obtained, which is dependent on the beam loading and the cavity detuning ($R >> R_L$):

$$P_f = ((1 + 2R_L \cdot I_{b0} \cdot \cos\varphi/|\mathbf{V_c}|)^2 + (\Delta\omega/\omega_{1/2} + 2R_L \cdot I_{b0} \cdot \sin\varphi/|\mathbf{V_c}|)^2) \cdot |\mathbf{V_c}|^2/8R_L.$$

The calculated power attains the minimum value equals beam loading power $P_b = |\mathbf{V_c}| \cdot I_{b0} \cdot \cos\varphi$ for $\tan\varphi = \Delta\omega/\omega_{1/2}$ and for depended on coupling the optimum loaded quality factor $Q_L = |\mathbf{V_c}|/(2I_{b0} \cdot \rho \cdot \cos\varphi)$.

7. CONTROL SYSTEM MODELING.

The objective of the control system is to stabilize the cavity voltage with the minimal power consumption supplied by the klystron. The feedback control system supported by feed-forward is modeled by the signal-flow graph according to figure 6. The cavity is represented by the transfer function $Z(\mathbf{s})$ linking the current and voltage (I, Q) components. Superposition of the feed-forward signal and beam loading current drive directly the cavity. Voltage (I, Q) components are compared to the reference vector **R** delivered by the set-point. The error signal is amplified by the proportional controller with gain G closing the feedback loop. Transducers K_1 and K_2 adapt signals to match its value for the controller and cavity environment.

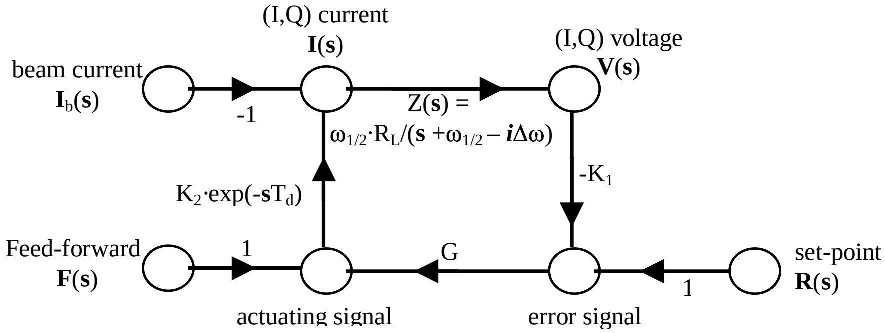

Figure 6. Signal-flow graph for cavity control system.

The cavity voltage is stated as follows:

$$\mathbf{V}(\mathbf{s}) = (K_2 \cdot \exp(-\mathbf{s}T_d) \cdot (G \cdot \mathbf{R}(\mathbf{s}) + \mathbf{F}(\mathbf{s})) - \mathbf{I}_b(\mathbf{s})) \cdot Z(\mathbf{s})/(1 + K \cdot G \cdot Z(\mathbf{s}) \cdot \exp(-\mathbf{s}T_d)),$$

where transducer ratio $K \equiv K_1 \cdot K_2$, loop delay $\equiv T_d$.

The main reason of the cavity voltage destabilization is the resonance frequency changing caused by micrphonics and Lorentz force detuning. The feedback system stabilizes the cavity voltage at the expense of an additional forward power. The high controller gain for the better accuracy and fast response is limited by the stability of the feedback system. The loop phase shift is the reason of the potential instability. The time delays caused by the wave-guide and the digital controller latency are important contributions to the phase shift in the closed loop of the feedback system. The detuned cavity behavior for driving step current $\mathbf{i} = 16$ mA ($i_g = 8$ mA, i_b=0, without feed-forward) is presented in figure 7 for the following parameters: loaded quality factor $Q_L = 3 \cdot 10^6$, characteristic resistance ρ =520 Ω (R_L = 1560 MΩ, $f_{1/2}$ = 217 Hz), detuning $\Delta\omega$ = 400 Hz, loop delay = 4 μs, proportional controller gain range G = 20 : 110.

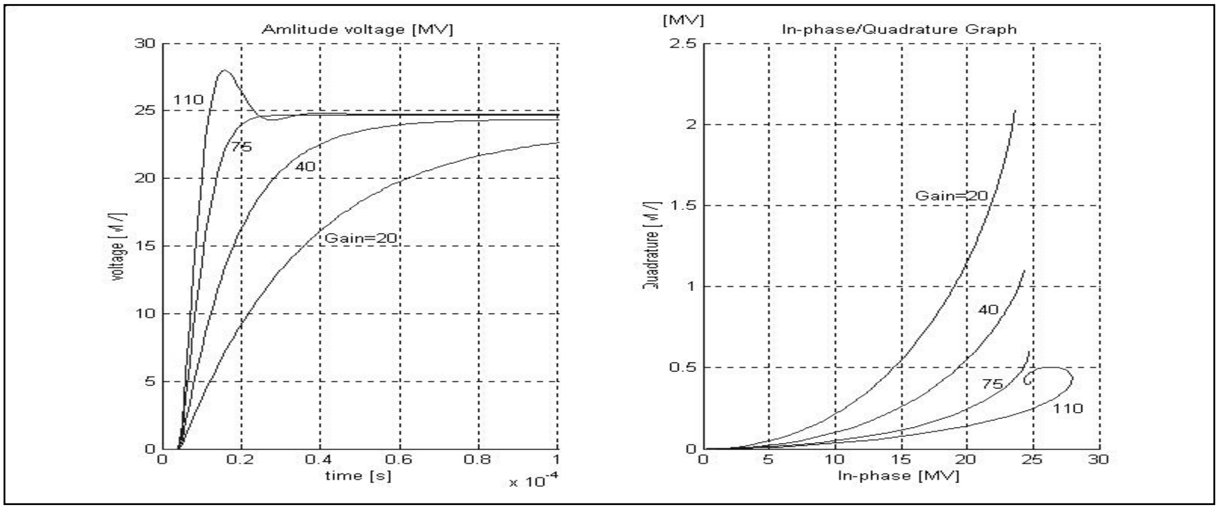

Figure 7. Cavity step response and I/Q graph for different controller gains (detuning = 400Hz, loop delay = 4 μs).

8. ELECTROMECHANICAL MODELING.

The super-conducting resonator has an extremely high loaded quality factor $Q_L = 3 \cdot 10^6$ and the narrow bandwidth of about 430 Hz (FWHM). The cavity is very sensitive to the mechanical distortion caused by microphonics and Lorentz force changing the resonator's frequency. In addition to the feedback control loop, the adaptive feed forward is applied to suppress the influence of the beam loading and the dynamic Lorentz force detuning. Furthermore the local piezo-translator system is developed to cope with the mechanically caused detuning of the cavity.

The cavity electrical model is based on the state space equation for the cavity voltage vector (v_r, v_i) driven by the current vector (i_r, i_i)

$$dv_r/dt = -\omega_{1/2} \cdot v_r - \Delta\omega \cdot v_i + \omega_{1/2} \cdot R_L \cdot i_r$$
$$dv_i/dt = \Delta\omega \cdot v_r - \omega_{1/2} \cdot v_i + \omega_{1/2} \cdot R_L \cdot i_i$$

The time-variant electrical model describes the cavity field as the function of the drive signal (generator + beam current) and non-stationary detuning $\Delta\omega$. It consists of the decomposed state space model with the non-linear vector function and integrator. The detuning $\Delta\omega$ is the additional external signal obtained from the output of the mechanical model. Due to the Lorentz force the cavity resonance frequency changes dynamically with square of the rising field gradient. The mechanical model describes the dynamic Lorentz force detuning which is the function of the time varying gradient. It is based on the state space representation for the mechanical modes of the cavity with the resonance frequency f and the mechanical time constant τ for each mode according to equation

$$d\dot{\Delta\omega}/dt = -(\dot{\Delta\omega})/\tau - (2\pi f)^2 \cdot (\Delta\omega + 2\pi \cdot K v^2),$$

where detuning $\Delta\omega$ and its derivative $\dot{\Delta\omega}$ are two phase-variables states for each mode. The mechanical model is driven by square of the cavity field gradient v^2 with Lorentz force detuning constant K. Three dominating resonance frequencies are considered in the model and the superposition of all modes yields the resultant detuning.

The detuning response of the mechanical model for driving step voltage **v** = 25 MV and voltage response of the electromechanical model for driving step current **i**=16 mA (i_g=8 mA, i_b=0) is presented in figure 8 for the following mechanical parameters: resonance frequencies: (280, 340, 420) [Hz], quality factor = 100 for each mode, Lorentz force detuning constants: (0.4, 0.3, 0.2) [Hz/(V/m)^2]. The mechanical model is weekly damped but the electromechanical model appears unstable!

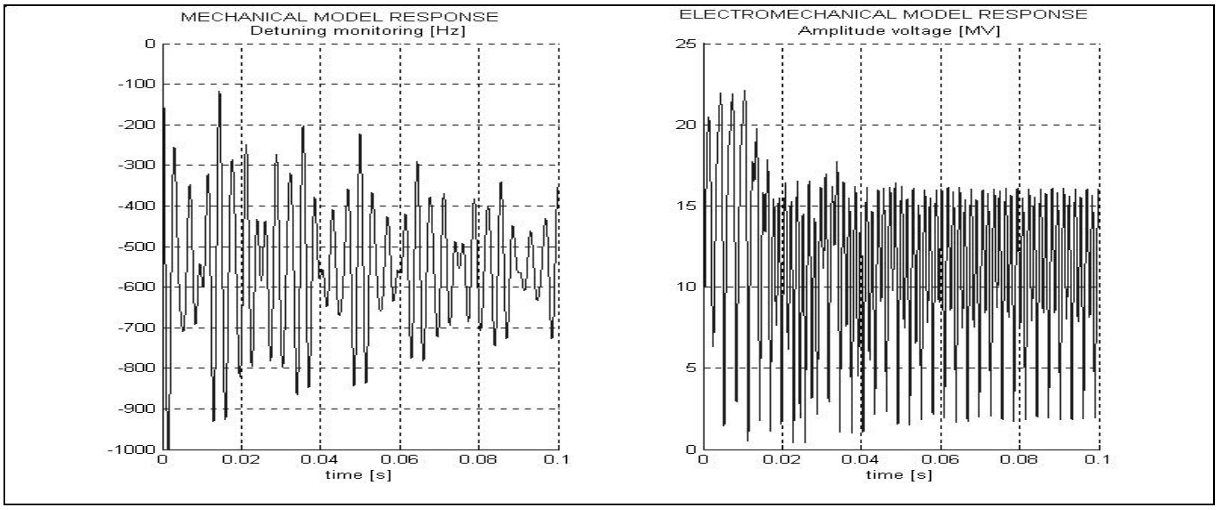

Figure 8. Cavity step response for mechanical and electromechanical model.

9. SIMULINK MODELING.

The model of the cavity control system is composed applying Simulink – dynamic system simulation for Matlab according to figure 9. The cavity is represented by its decomposed non-linear state space electrical model and the mechanical part as a linear state space model generating the non-stationary detuning. The required amplitude and phase set-points converted to the in-phase (*Re*) and quadrature (*Im*) components are compared to the actual (I,Q) cavity voltage. The proportional controller as a matrix gain amplifies the signal error and closes the feedback loop. The amplitude and phase of the beam loading current and feed-forward signal are adequately generated and converted to its (I,Q) components. The transducers match properly signals level to the controller circuit and the cavity environment.

For the simulation purpose the initializing Matlabs' m.file is used to control the model implemented by the Simulink Toolbox. The main parameters of the model are available from this file and are combined in the table below.

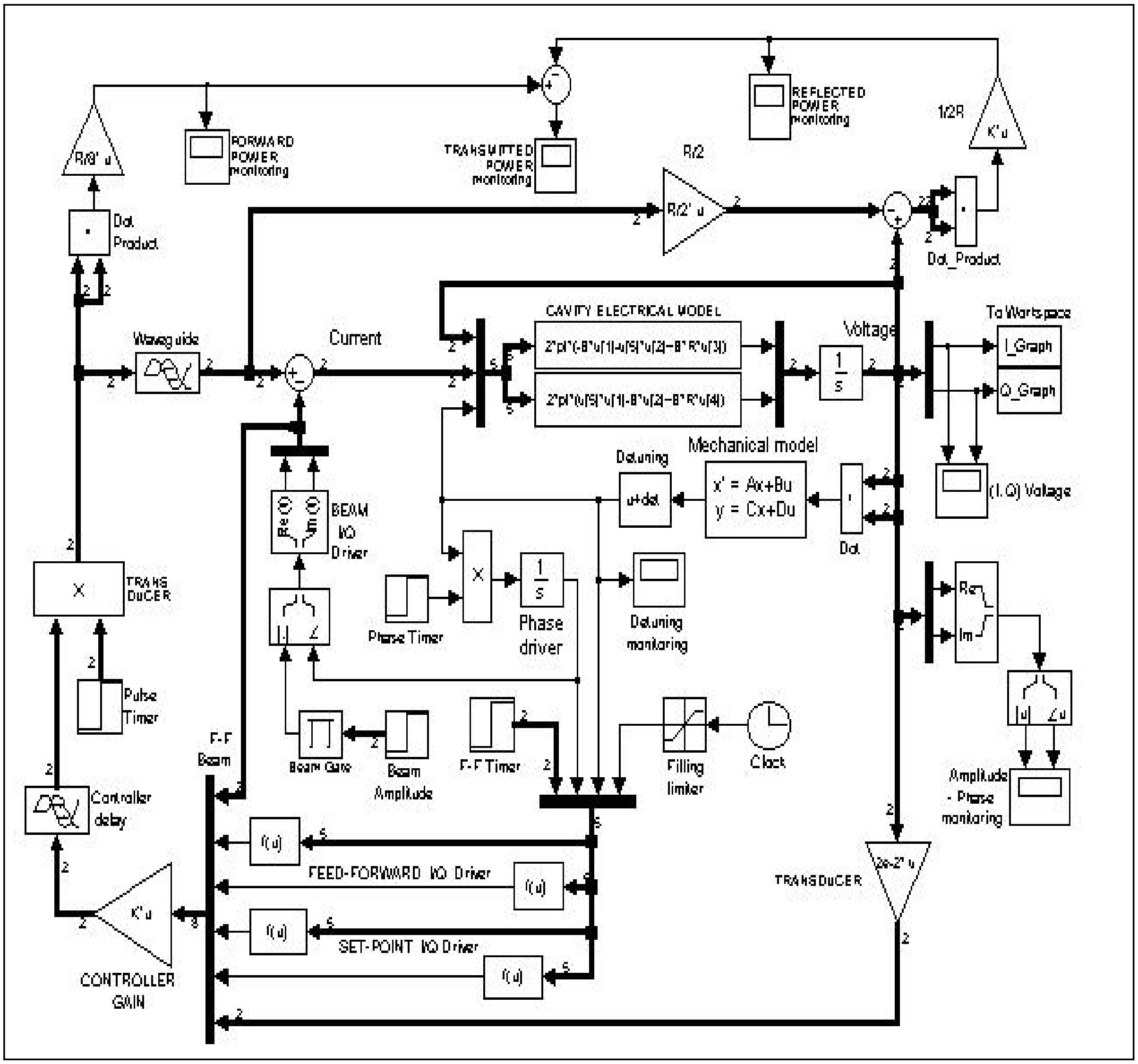

Figure 9. Simulink model of the cavity control system.

CAVITY ELECTRICAL parameters	SIGNAL parameters
f_0 = 1300 …………………resonance frequency [MHz] ρ = 520 …………………..characteristic resistance [Ω] $Q_L = 3 \cdot 10^6$………………………..loaded quality factor $R_L = Q_L \cdot \rho$ = 1560………………..load resistance [MΩ] $f_{1/2} = f_0/2Q_L$ = 216……………….half band-width [Hz] Δf = 428 ……………………………pre-detuning [Hz]	Beam = 8........………beam current average value [mA] V = 25……………..amplitude of cavity voltage [MV] φ = -0.1……………………phase of cavity voltage [rad] Φ_0 = -1.4……………initial phase for integrator [rad] $T_f = \ln 2/2\pi f_{1/2}$……………..…………..filling time [s] T = 1.3e-3……………………..pulse generator time [s]
CAVITY MECHANICAL modes parameters	**CIRCUIT parameters**
f = [280, 340, 420]……resonance frequencies vector [Hz] **τ** = [100/**f**(1),100/**f**(2),100/**f**(3)]…time constants vector [s] **K** = [0.4, 0.3, 0.2]……Lorentz force detuning constants vector [Hz/(V/m)^2]	FF…………….Feed-forward: on – FF = 1; off – FF = 0 G = 75………………………………controller gain del = 1e-6……………………..wave-guide delay [s] lat = 4e-6...DSP latency [s]

10. CAVITY FEED-FORWARD AND FEEDBACK OPERATION.

In the real operational condition the cavity is driven in pulse mode forced by the control feedback supported by feed-forward. During the first stage of the operation the cavity is *filling* with constant forward power resulting in an exponential increase of the electromagnetic field according to its natural behavior in the resonance condition. When the cavity gradient has attained the half of the final value and the transmitted power has reached the value of the forward power (no reflection) (figure 5) the beam loading current is injected resulting in the steady-state *flattop* operation. Turning off both generator and beam current yields an exponential *decay* of the cavity field.

Considering the deterministic conditions of the operation the cavity can be driven directly by the determined current resulting in the required cavity behavior. For repetitive beam loading and cyclic dynamic detuning the adaptive feed-forward signal is applied. To accomplish the cavity voltage **V** required at the *flattop* level the *filling* feed-forward signal can be estimated as the phase modulated current step

$$\mathbf{i}_f(t) = 2|\mathbf{V}|e^{i\varphi}/R_L \cdot \exp(i\Phi(t)),$$

so that, magnitude |**V**| stands for the half of the final exponentially increasing amplitude of the cavity voltage, phase φ shifted to the beam. The appropriate phase modulation $\Phi(t)$ compensates the cavity detuning according to the relation

$$d\Phi/dt = \Delta\omega,$$ so that RF tracks the cavity resonance frequency.

Therefore detuning $\Delta\omega$ obtained from the output of the mechanical model drives the integrator, which establishes the phase condition for the generator and the beam. The feed-forward and set-point I/Q drivers additionally shift the phase Φ by determined value φ related to the beam loading. The accurate initial condition $\Phi_0 = \Phi(0)$ of the integrator adjusts the required *flattop* value $\Phi_f = \Phi(T_f)$ for *filling* time T_f.

At the beam injection instant $T_f = \ln 2/\omega_{1/2}$ the phase and amplitude of the cavity voltage are set stable as the result of the steady state forced in the *flattop* operation. For the required condition with the beam loading $\mathbf{i}_b = |\mathbf{i}_b| \cdot \exp(i\Phi_f)$ and the detuned cavity ($\Delta\omega \neq 0$) the driving *flattop* feed-forward current can be estimated as follows:

$$\mathbf{i}_f = (|\mathbf{V}|e^{i\varphi} \cdot (1 - i\Delta\omega/\omega_{1/2})/R_L + |\mathbf{i}_b|) \cdot \exp(i\Phi_f),$$ so that predicted component = $\mathbf{i}_b$ compensates the beam loading current.

The only feed-forward cavity voltage amplitude response and power monitoring are presented in figure 10 for the cavity parameters according to the table from chapter 9.

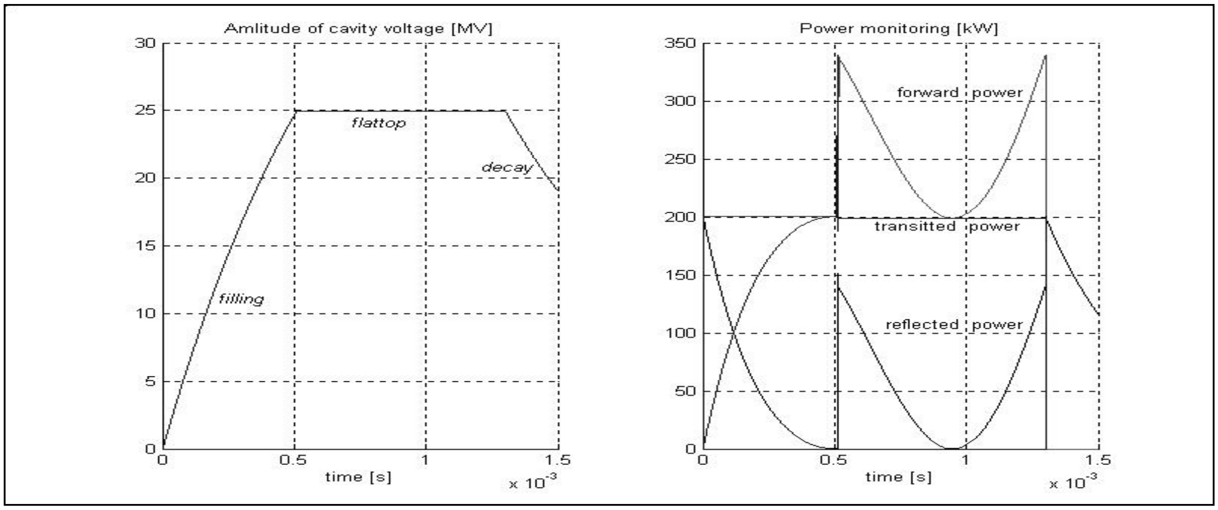

Figure 10. Feed-forward cavity voltage amplitude response and power monitoring.

During *flattop* operation the dynamic Lorentz force still decreases the cavity resonance frequency. The cavity voltage is stable at the expense of an additional power reflected due to the mismatch caused by the cavity detuning. To minimize the required peak power the time varying detuning is shifted by the proper cavity pre-detuning, so that forward power is symmetrically extended along the *flattop* range. The appropriate initial phase Φ_0 adjusts the cavity *flattop* phase $\Phi_f + \varphi$. The cavity voltage phase and the cavity detuning are presented in figure 11 for Φ_0 = -1.4 rad, Φ_f = 0, φ = -0.1 rad and pre-detuning $\Delta\omega$ = 451 Hz.

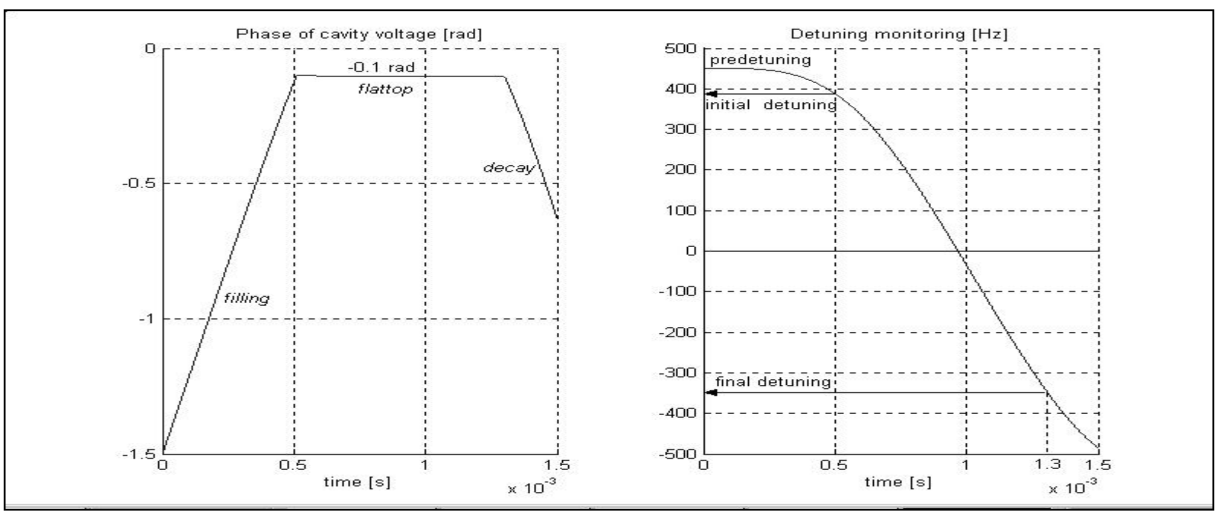

Figure 11. Feed-forward cavity voltage phase response and detuning monitoring.

The required cavity voltage can be accomplished through the feedback operation with an accuracy rely on the limited controller gain. Therefore applying adaptive feed-forward the main task for the feedback is to suppress stochastic errors.

The *filling* set-point for the feedback loop is estimated as the phase modulated exponentially rising signal:

$$\mathbf{r}(t) = 2|\mathbf{V}|e^{i\varphi}/R_L\cdot(1- \exp(-\omega_{1/2}t))\cdot\exp(i\Phi(t)).$$

After *filling* time $T_f = \ln2/\omega_{1/2}$ the *flattop* level $\mathbf{V} = |\mathbf{V}|e^{i\varphi}\cdot\exp(i\Phi_f)$ is established.

The only feed-back performance for amplitude, phase, detuning and forward power is presented in figure 12 for different levels of the cavity voltage. For any case the cavity pre-detuning $\Delta\omega$ and initial phase Φ_0 are adequately set up.

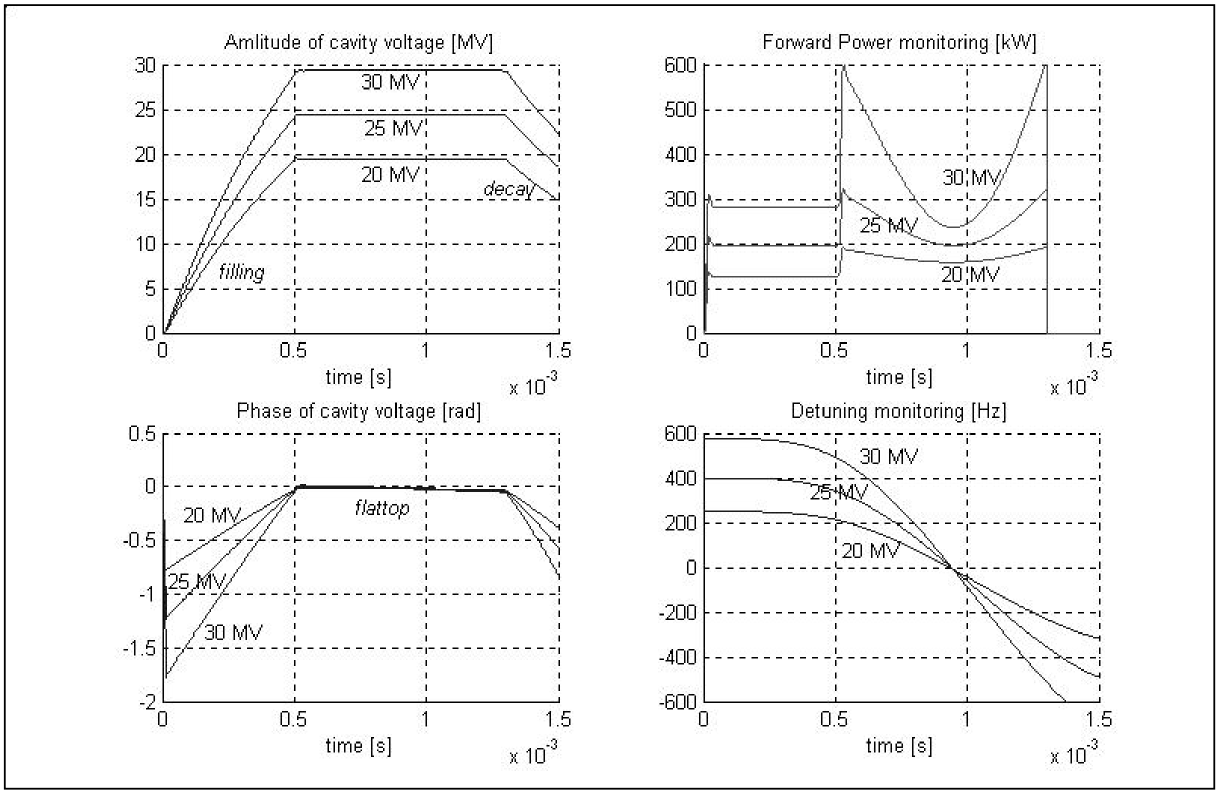

Figure 12. Cavity voltage amplitude, phase, detuning and power monitoring for feed-back operation only.

11. VECTOR SUM CONTROL.

In the TESLA collider the control system stabilizes the vector sum of pulsed accelerating fields in 32 cavities powered by a single 10 MW klystron. For the given parameters and signal requirements the optimal operational condition can be evaluated for a single cavity. For identical cavities the RF power should be distributed evenly to individual resonators therefore the signal and power results are the same like for a single cavity. Due to variation and spread of parameters the proper averaging procedure is required to optimize performance of the system. In special cases (e.g. cavity quench) it is desirable to operate cavities at different gradients to maximize energy gain of a system with multiple resonators. The system of multiple cavities can be modeled as the serial connection of resonators with diverse parameters and individual appropriate power distribution. The only common signal **i** drives many cavities and should be properly distributed to individual resonators in respect to the amplitude and phase requirements. For the given beam loading current $\mathbf{i}_b$ common for all cavities and for the required voltage $\mathbf{V}_k$ of the k-cavity the driving current for the steady state in the resonance condition is as follows

$$\mathbf{i}_k = \mathbf{V}_k/R_k + \mathbf{i}_b = \mathbf{p}_k \cdot \mathbf{i}.$$

The relative complex coefficient $\mathbf{p}_k$ of the signal distribution for k-cavity can be evaluated taking as a reference the average cavities current $\mathbf{i} = (\mathbf{V}/R)_{ave} + \mathbf{i}_b$, therefore

$$\mathbf{p}_k = (\mathbf{V}_k/R_k + \mathbf{i}_b)/(\mathbf{V}/R)_{ave} + \mathbf{i}_b) = (\mathbf{V}_k + R\mathbf{i}_b)/(\mathbf{V}_{ave} + R\mathbf{i}_b)$$ for nominally identical cavities, where

$\mathbf{V}_{ave}$ – average cavities voltage as the required vector sum divided by the number of cavities, $R_k = R = Q_L \cdot \rho$ – load resistance for any cavity.

The Simulink implementation of the control system for the 8-cavities section is presented in figure 13 in relation to the functional block diagram. The cavities cryomodule subsystem outputs 8 voltage vectors and detuning scalars, after that which are summarized and averaged. According to the amplitude and phase requirements the I/Q drivers subsystem generates the required in-phase, quadrature components for the beam, feed-forward and set-point signals applying additionally the information of the average resonator detuning. The feedback and feed-forward operation can be proceeded independently. According to the signal requirements for every cavity the common driving current is adequately distributed to the resonators. The monitoring subsystem registers and presents the information for every cavity - voltage vector in amplitude, phase representation, power, detuning as well as the average value of each quantity.

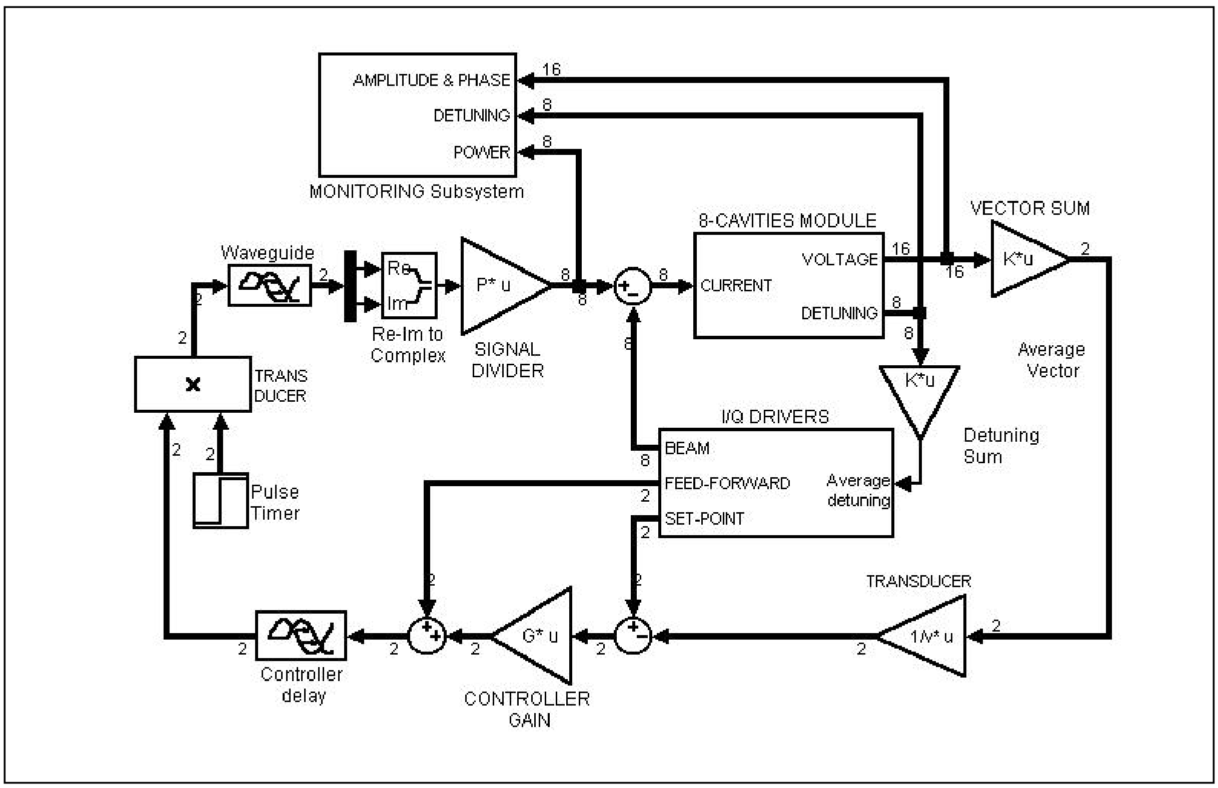

Figure 13. Simulink model of 8-cavities cryomodule control system.

The eight cavities parameters collected in the table below are considered for the simulation purpose.

Cavity	1 ÷ 4	5 ÷ 8	variation
Gradient [MV/m]	12	20	± 10%
Phase [˚]	0	-10	± 5˚
Quality factor [10^6]	2	2	± 10%
Lorentz force detuning const. vector [$Hz/(V/m)^2$]	(0.3, 0.3, 0.3)	(0.3, 0.3, 0.3)	± 10%
Mechanical resonance frequency vector [Hz]	(280, 350, 440)	(280, 350, 440)	± 10%
Mechanical quality factor vector	(100, 100, 100)	(100, 100, 100)	± 10%

The signal requirements are just fulfilled in the resonance as well as in the steady detuning condition for the operation without beam. The simulation results for amplitude, phase, forward power and detuning are presented in figure 14.

During the operation with the Lorentz force detuning without beam the average vector is stabilized but particular cavities phases are strongly deviated. The simulation results for amplitude, phase, forward power and detuning are presented in figure 15.

During the operation in the resonance condition and with 8 mA average value beam the average vector is stabilized but the particular cavities vectors strive for its final *flattop* values. The simulation results for amplitude, phase, forward power and detuning are presented in figure 16.

During the operation with the Lorentz force detuning and with 8 mA average value beam the average vector is stabilized but the particular cavities amplitudes strive for its final *flattop* values and phases diverge from its common point. The simulation results for amplitude, phase, forward power and detuning are presented in figure 17.

Taking into account the variation of the cavities signals and parameters four series of simulations has been carried out with the Lorentz force detuning and with 8 mA average value beam.

1. Variation ±10% of the cavities voltage amplitude (required phases are established) when the cavities parameters are stable at the nominal values according to the table above (figure 18).

2. Variation ± 5˚ of the cavities voltage phases (required amplitudes are established) when the cavities parameters are stable at the nominal values according to the table above (figure 19).

3.and 4. Variation ±10% of the cavities parameters: electrical quality factor (figure 20) and mechanical parameters (figure 21) when the required cavities signals are established at the nominal values according to the table above.

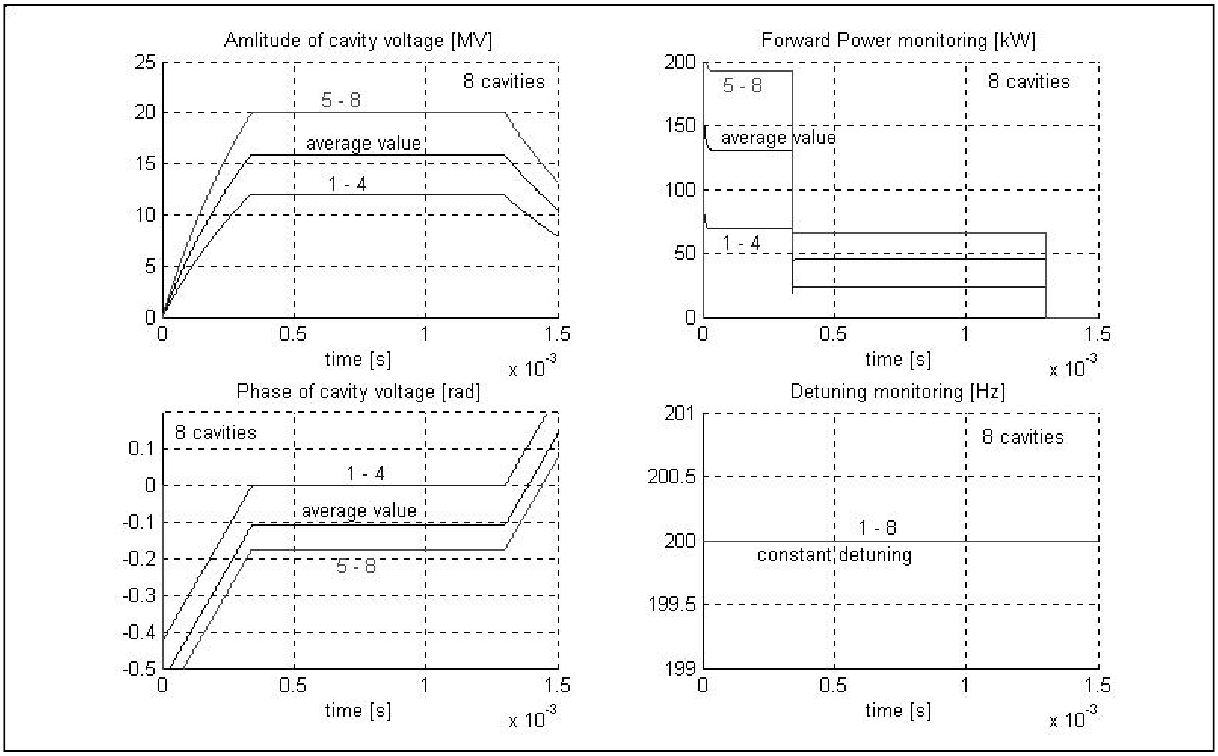

Figure 14. Simulation results of 8 cavities operation for stable detuning without beam.

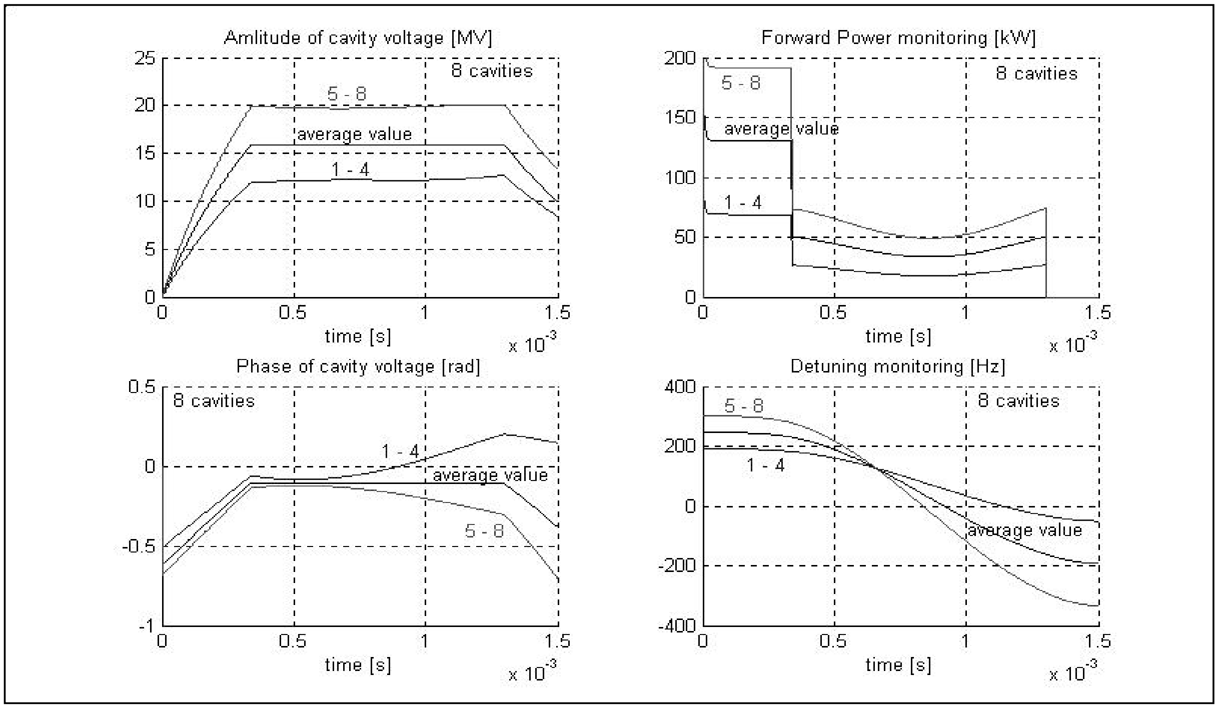

Figure 15. Simulation results of 8 cavities operation for dynamic Lorentz force detuning without beam.

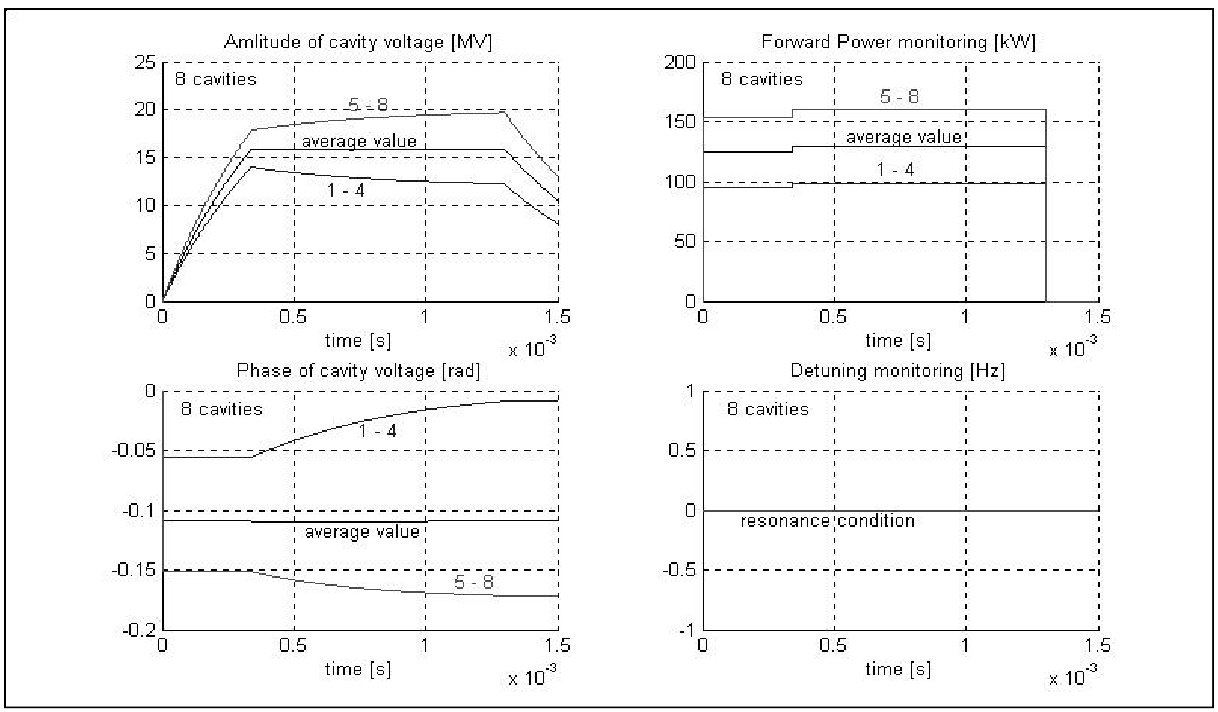

Figure 16. Simulation results of 8 cavities operation in resonance condition with beam.

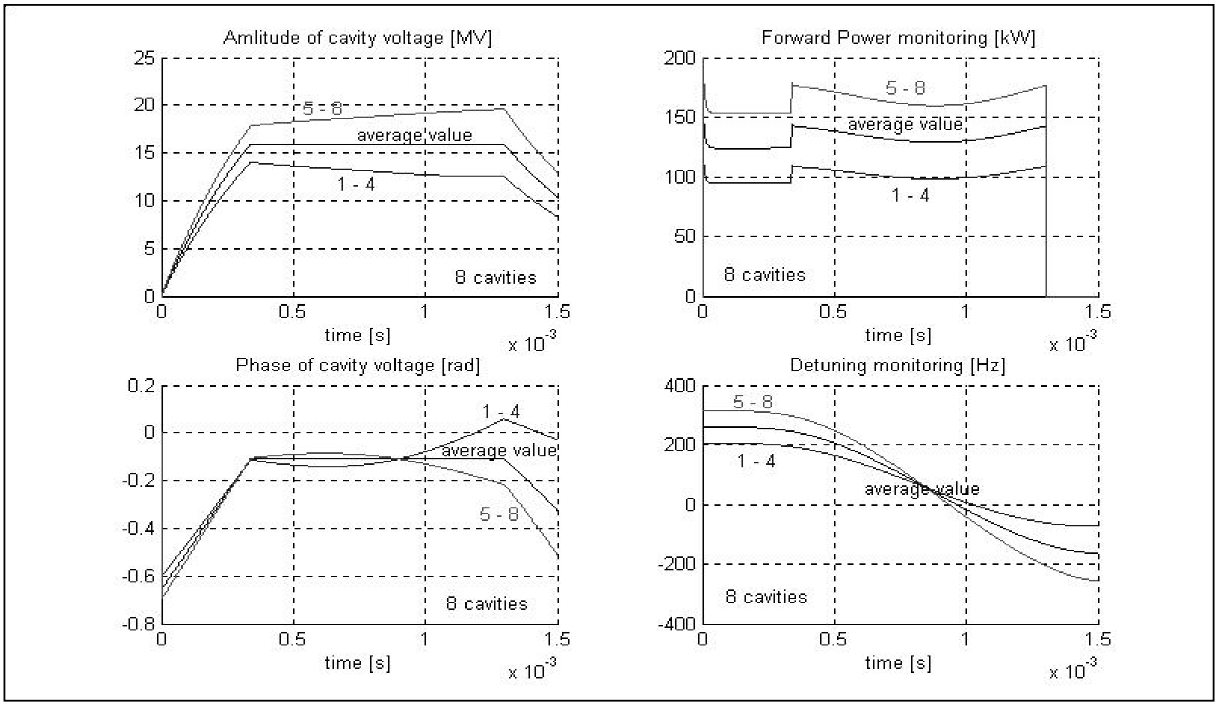

Figure 17. Simulation results of 8 cavities operation for dynamic Lorentz force detuning with beam.

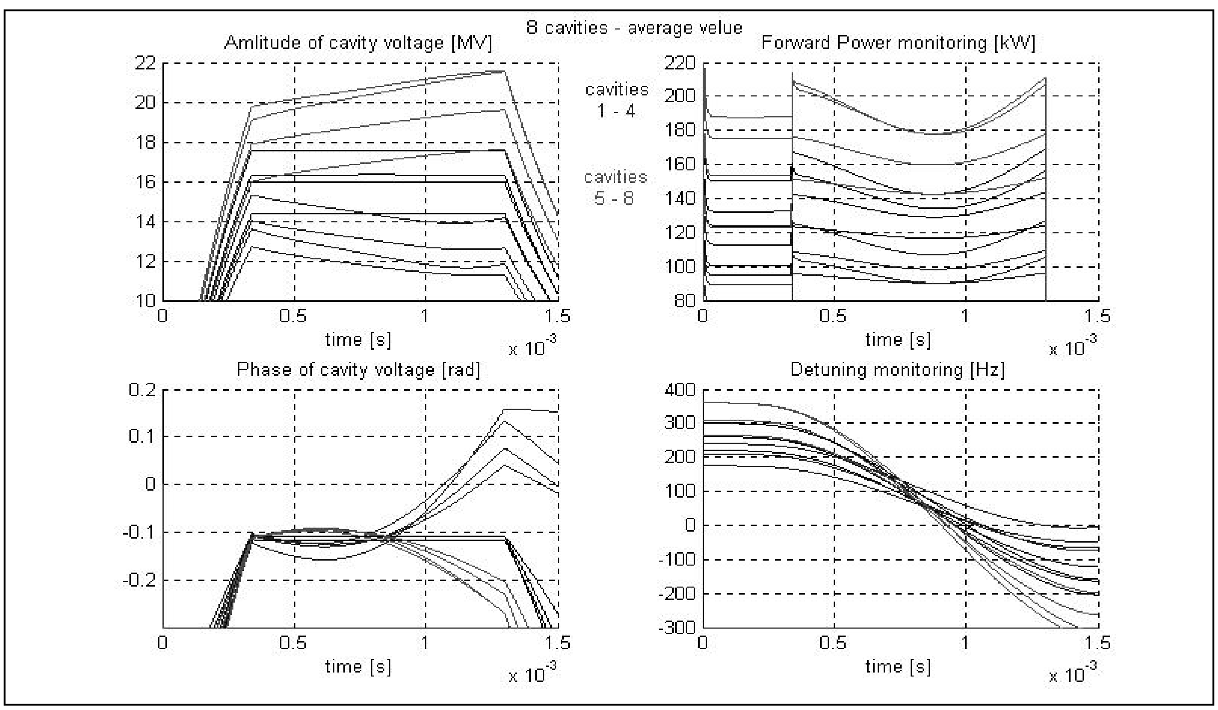

Figure 18. Variation ±10% of cavities voltage amplitude when cavities parameters are stable.

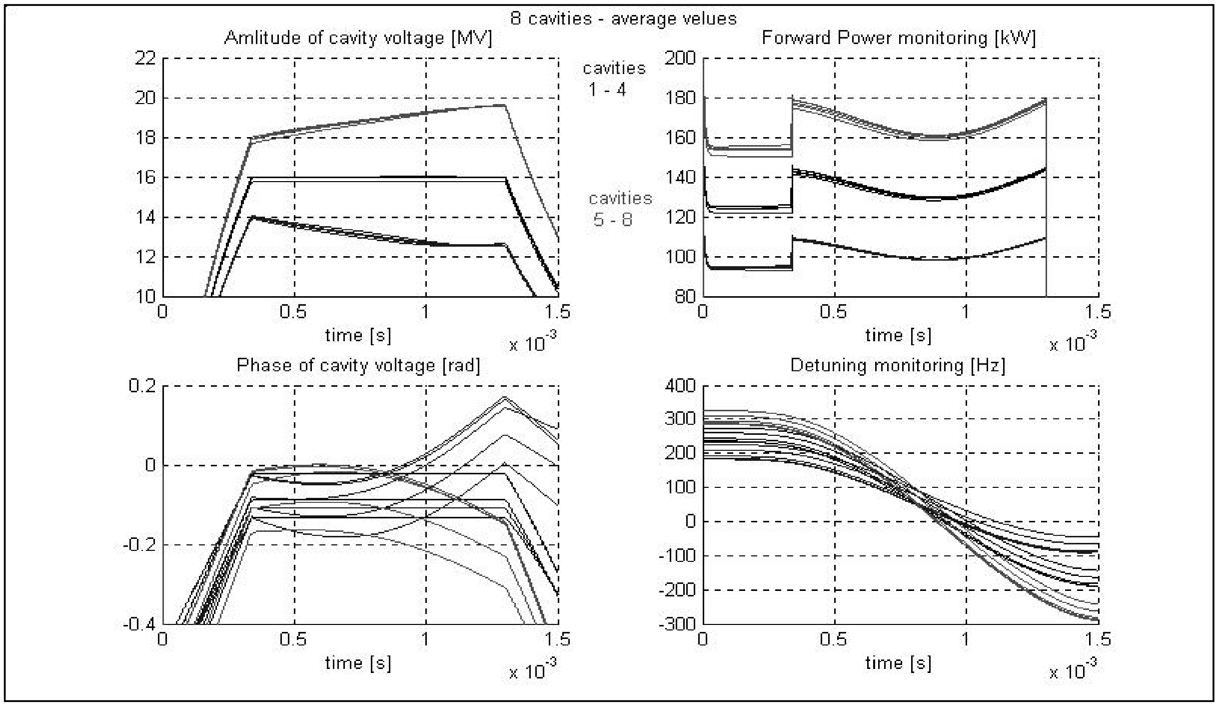

Figure 19. Variation ±5˚ of cavities voltage phases when cavities parameters are stable

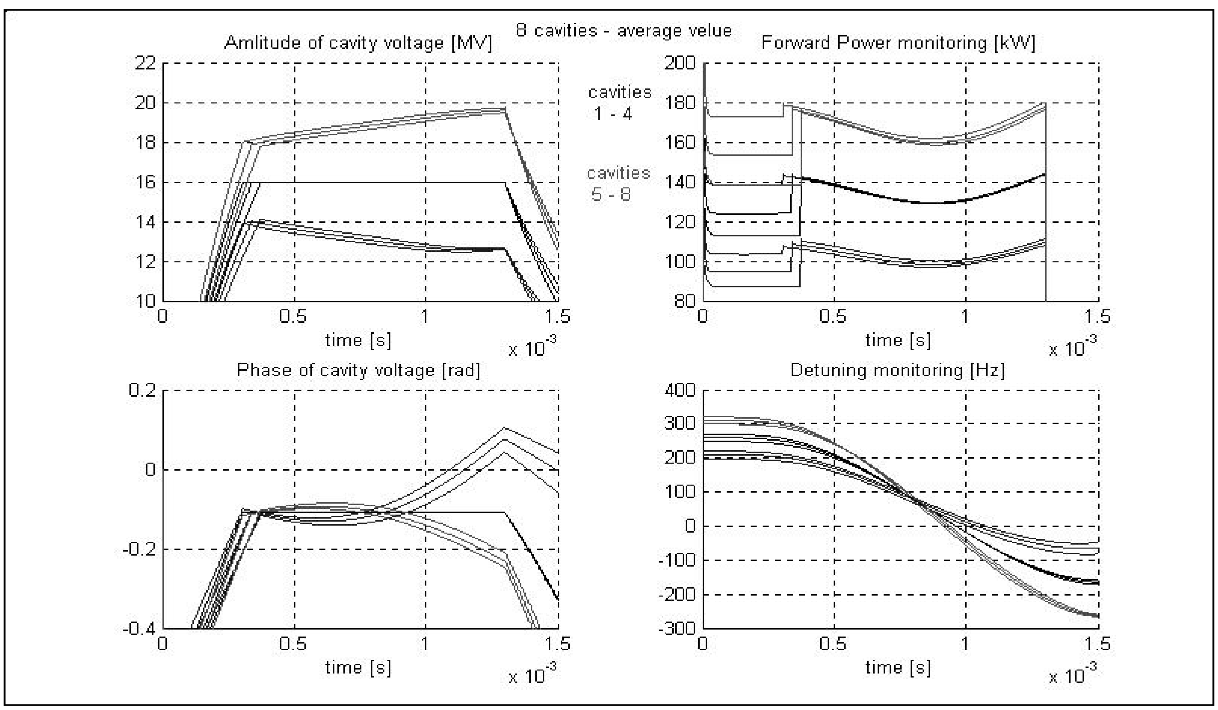

Figure 20. Variation ±10% of electrical quality factor only when required cavities signals are established.

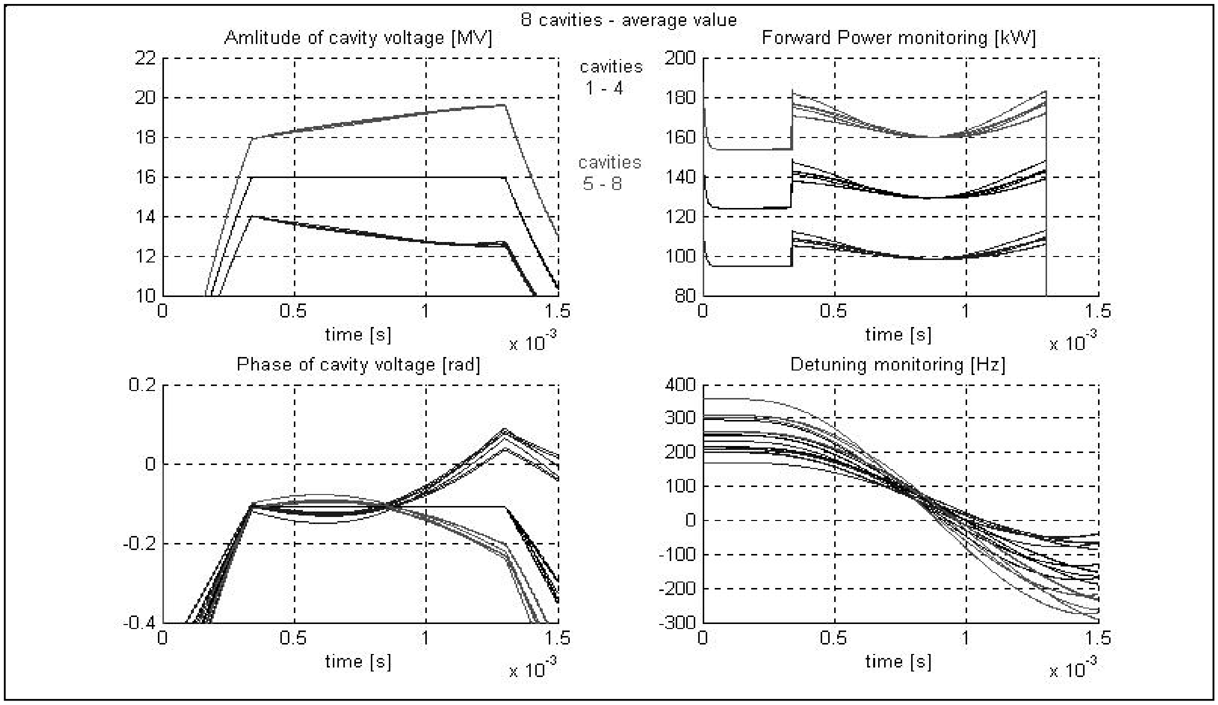

Figure 21. Variation ±10% of mechanical parameters only when required cavities signals are established.